Alexandra Park Primary School
"Care, Aspire, Achieve"

Teaching MATHS

Alexandra Park Research School

Supported by the Education Endowment Foundation

FUNDAMENTALS → ELEMENTS → TECHNIQUES

APPLICATION

CHILDREN ACTIVELY INVOLVED AT EVERY STAGE

FACILITATION

To guide, mentor, deliver knowledge and enable pupils to succeed.

SMALL STEPS FOLLOWING SHANGHAI MATHS APPROACH

MODELLED PROCESSES THINKING ALOUD

DIRECT INSTRUCTION

To develop understanding of the subject so that children acquire powerful knowledge through specific teaching.

SPECIFIC DELIVERY OF CORE KNOWLEDGE

ADAPTIVE TEACHING LOWEST 20%

SLOW RELEASE

To use models, think alouds, paired and independent opportunities so that pupils know how to apply knowledge and skills appropriately.

SPACED RETRIEVAL PRACTICE

ACCESSIBLE NUMBERS

EARLY ACCESS

To use specific questions, which draw upon prior knowledge connected to new learning, allowing all children to have success at the start of a lesson.

RETRIEVAL FOCUS

COLLABORATIVE AND PURPOSEFUL MATHS TALK

REASONING FOR ALL

Include reasoning and hedging opportunuties throughout lessons to provide all children with real verbal reasoning opportunities

REASONING CHALLENGES TO DEEPEN

SPECIFIC AND RELEVANT TO PROMOTE PARTICIPATION

QUESTIONING

To use cleverly constructed questioning, which prompt pupils to think deeply and carefully, and create opportunities to check depth of understanding.

DIAGNOSTIC TO ESTABLISH ASSESSMENT

CORRECT MATHEMATICAL LANGUAGE

ORACY

To use the oracy framework, talk tactics and associated stems to allow ALL pupils to engage in meaningful, well-informed discussions.

EFFECTIVE VERBAL REASONING

ELEMENTS

DIRECT:

DIRECT AND SPECIFIC USING BOTH VERBAL AND WRITTEN

INDIRECT:

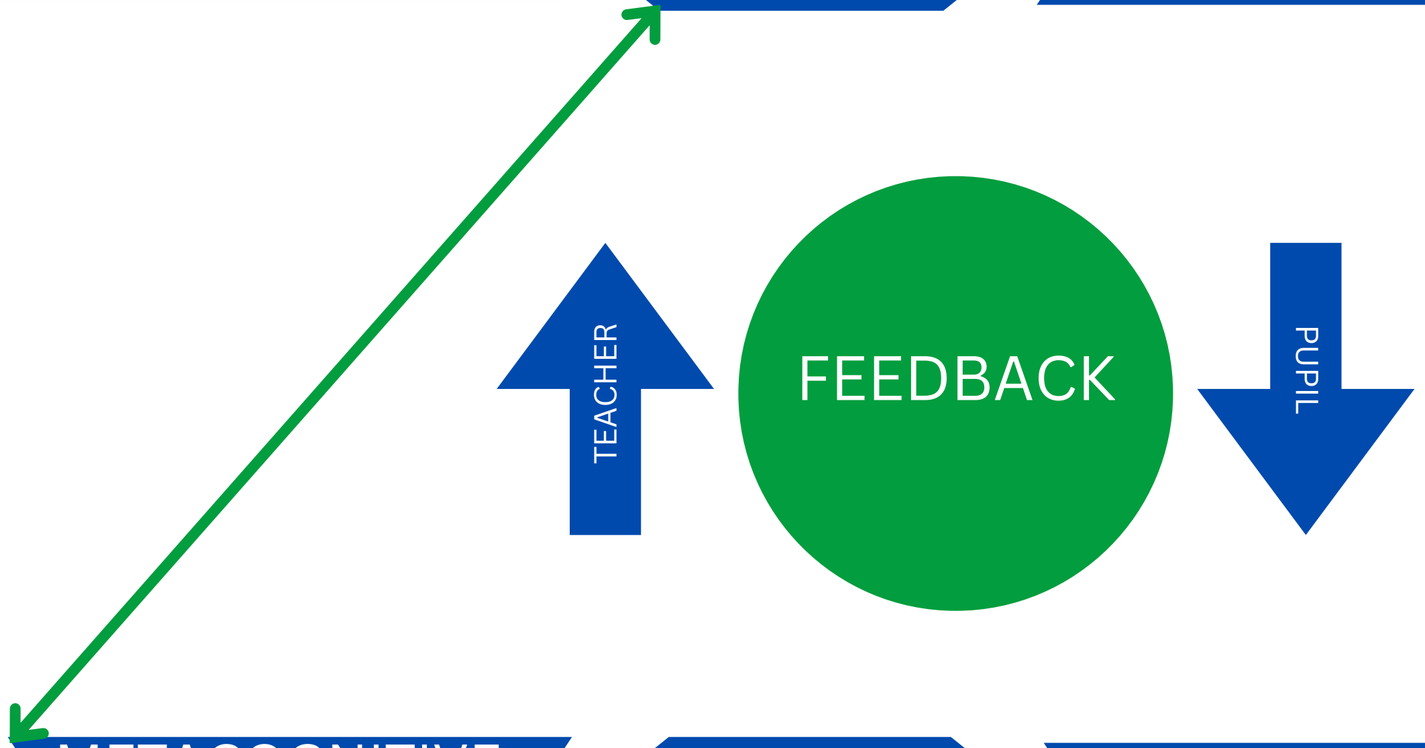
PUPILS INFORMED THAT SOME OF WHAT THEY HAVE DONE IS INCORRECT - ONUS ON THEM TO FIND THE ERROR AND CORRECT

IN AND OUT:

TIMELY FEEDBACK WITHIN CLASS, ALLOWING PUPILS TO RESPOND. APPROPRIATE FEEDBACK OUT OF CLASS, PROVIDING STRUCTURED OPPORTUNITY FOR ACHIEVEMENT

WHOLE CLASS:

PROVIDE WHOLE CLASS GENERAL FEEDBACK ON KEY THEMES ASSESSED FROM PREVIOUS LESSONS AND LEARNING OPPORTUNITIES



METACOGNITIVE:

CHILDREN UTILISE THE MONITOR AND EVALUATE STEPS OF METACOGNITION TO ASSESS THEIR OWN WORK - THIS CAN BE DONE WITH THE HELP OF THE TOOLKIT

PAIRED:

PUPILS SUPPORT EACH OTHER WITHIN THE OPPORTUNITIES PROVIDED TO WORK COLLABORATIVELY, SHARING AND COMPARING THEIR WORK

CORRECT:

PUPILS USE THEIR OWN REFLECTIONS TO CORRECT THEIR WORK AND WRITE SHORT REFLECTION STATEMENTS AS A NOTE FOR FUTURE LEARNING

REFLECT:

PUPILS PROVIDED OPPORTUNITIES TO REFLECT ON THEIR OWN LEARNING, COMPARING THEIR WORK TO MODEL ANSWERS AND WORKING

ELEMENTS

PITCH:

QUESTIONS ARE APPROPRIATELY PITCHED TO GARNER THE RIGHT LEVEL OF THINKING

NESTING:

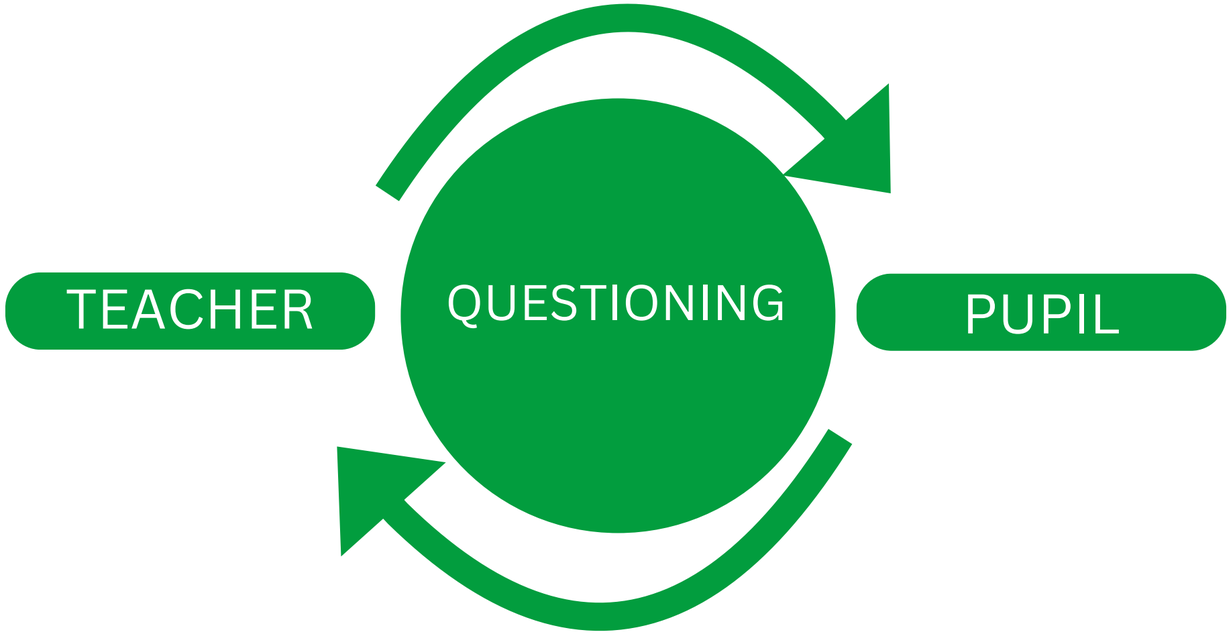
PUPILS PROVIDED TIME TO RECEIVE, CONSIDER AND PROCESS THE QUESTION BEING POSED

ORACY:

ESTABLISH AN ENVIRONMENT WHERE ASKING AND ANSWERING QUESTIONS BECOMES STANDARD AND CELEBRATES MISTAKES.

FOLLOW-UP

ENSURE QUESTIONS ARE ASKED TO FURTHER PROBE OR CLARIFY PUPILS' THINKING. USE THE POWER OF HOW AND WHY.



METACOGNITIVE:

CHILDREN USE THE PLAN STEP OF THE METACOGNITIVE PROCESS IN ORDER TO ORGANISE AND PREPARE THEIR RESPONSES - PUPILS AND TEACHER USE MONITOR AND EVALUATE TO REPOND APPROPRIATELY.

SHARE:

ALLOW TIME FOR SPECIFIC QUESTIONS TO BE DISCUSSED IN PAIRS OR AS A CLASS, ENCOURAGING PUPILS TO BUILD, CHALLENGE, CLARIFY OR PROBE

ALL:

USE TECHNIQUES WHICH ALLOW THE TEACHER TO GATHER A FULL RANGE OF RESPONSES THEREFORE GAINING A GREATER AWARENESS AND UNDERSTANDING OF THE WHOLE CLASS

STEMS

PROVIDE PUPILS WITH STEMS SO THEY KNOW HOW YOU WANT THEM TO RESPOND.

Alexandra Park Maths Lesson Design – Non-Linear 8-Tier Planning

CENTRAL QUESTIONS:

Am I secure with exactly what the LO is?

How best can the learning objective be achieved?

Planning Reflection 1 – Prior Knowledge

- What must already be secure?
- What needs recap, retrieval, or pre-teaching?
- If gaps persist → Tier 6 (Spotlight Children)
- Once secure → Return to Central Question
- What are the specific needs of my class?
- How are my lessons going to build on prior learning, and toward future learning?

Oracy & Language

Structured, purposeful mathematical talk.

- Which stem sentences are essential?
- Which vocabulary must be explicit?
- Where does 'My Turn, Your Turn' fit?
- Where would nesting, paired, trio or whole class discussion work?
- What Talk Tactics are pertinent at each point?

Planning Reflection 2 – Misconceptions

- What errors are likely?
- Why do they occur?
- Address explicitly before moving on.
- If understanding and language-based → (Oracy & Language)

Planning Reflection 3 – CPA & Representation

- Which concrete resources support understanding?
- How do pictorial models expose structure?
- If confusion arises → Return to Tier 2
- If secure → Move to Tier 4 (White Rose Questions) and Tier 5 (Depth and Adaptation)

Planning Reflection 4 – White Rose Specifics

- How do the teaching slides prepare them for the questions?
- Which questions directly serve the LO?
- What can be repeated or removed?
- What can be edited or changed?
- If secure → Move to Tier 5 (Depth & Adaptation)

Planning Reflection 5 – Depth of learning & Adaptation of lesson

- How can understanding be deepened?
- Can I include these activities as part of my input?
- Use prompts such as: Explain why..., Convince me..., What's the same/different?
- Use ChatGPT and ask it to create reasoning questions based around – True or False / Always, Sometimes, Never / Must, Could, Can't / Which one doesn't belong? (See prompt in notes section)
- Use ChatGPT and ask it to create an investigation based on the strict LO (See prompt in notes section)
- Challenge should not introduce new content and should still be linked to the LO

Planning Reflection 6 – Spotlight Children

- Who may struggle and why?
- What barriers exist (language, memory, representation)?
- Maintain the same Learning Objective where possible
- Routes → Tier 1 (Readiness) or Tier 3 (CPA & Representation)

Planning Reflection 7 – Games, Problems & Eureka moments

- Does this reward understanding, not speed?
- Is the content the same as the lesson?

Reasoning Mastery Challenge Principles

- ✔ Same content
- ✔ Open task
- ✔ Develops & Rewards understanding

Notes and Prompts

I am teaching Year _____ in the UK. I am teaching them _____ (be specific). I would like some reasoning activities along the lines of true or false, always sometimes never, which one doesn't belong and must could can't or questions that allow for hedging and generalisations. I need them to be accessible independently as a written reasoning activity after their procedural rehearsal of the content.

I am teaching Year _____ in the UK. I am teaching them _____ (be specific). I would like an extended investigation to allow them to both practice the key skill and deepen their understanding.

TECHNIQUES

APPLICATION

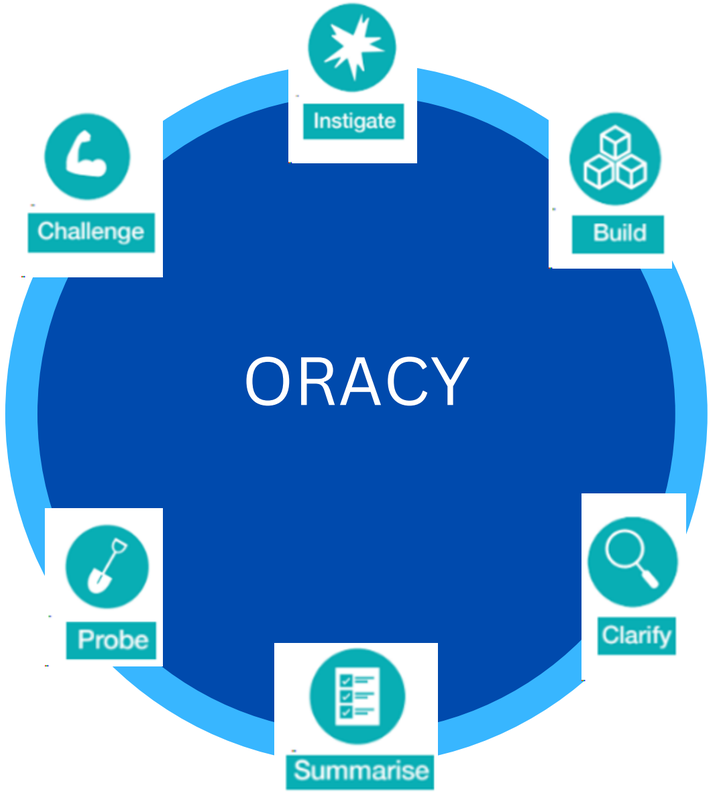
TEACH AND FACILITATE



WHAT HAPPENS WITHIN A LESSON? HOW?

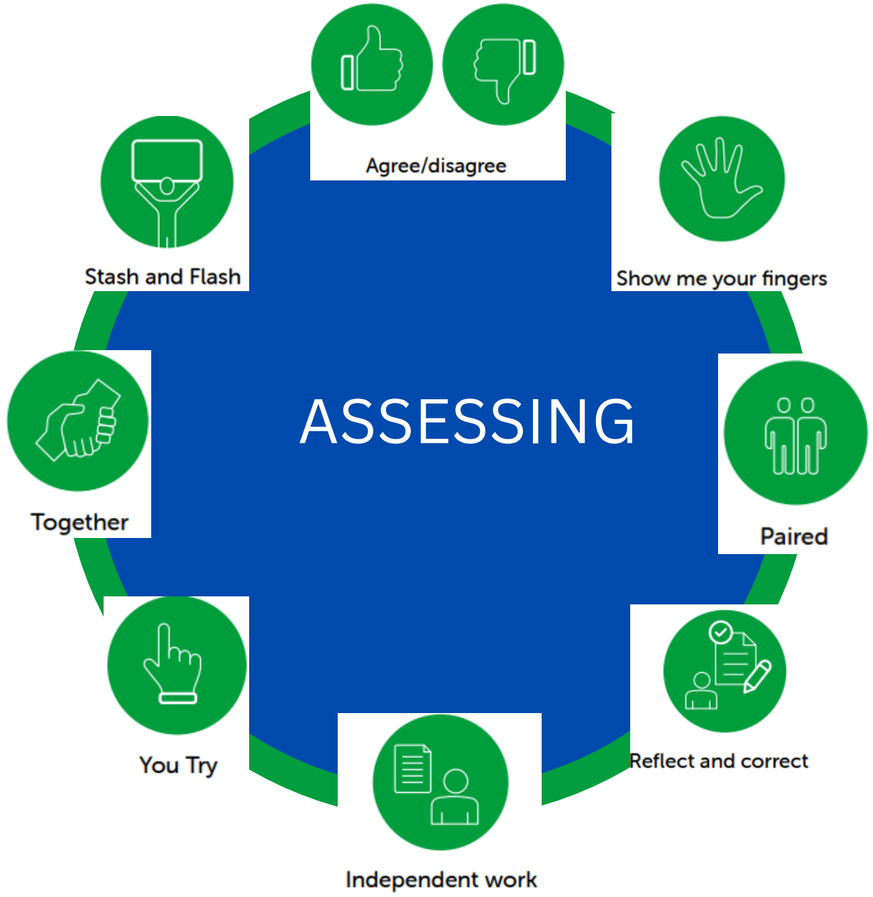


Aspire Challenges



Aspire Challenges

- Can you connect to me?
- Can you represent me in a different way?
- Can you put me in a word problem?
- Can you use me to create a true or false question?
- Can you explain how you solved me?

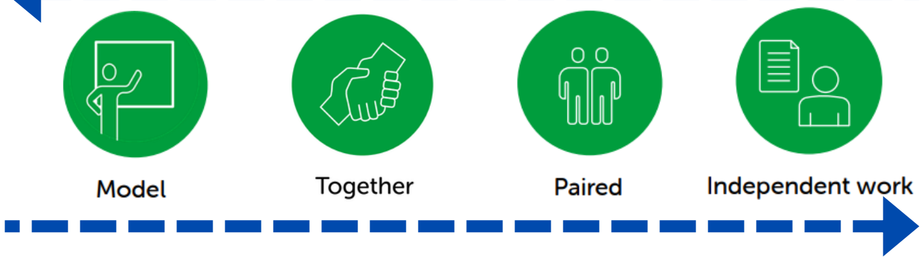


APPLICATION

Developing Reasoning and Problem Solving

HOW DO WE DO IT?

Different Approaches





True or False Reasoning Problems



Hedging



Goal-Free Problems



Worked examples: right and wrong

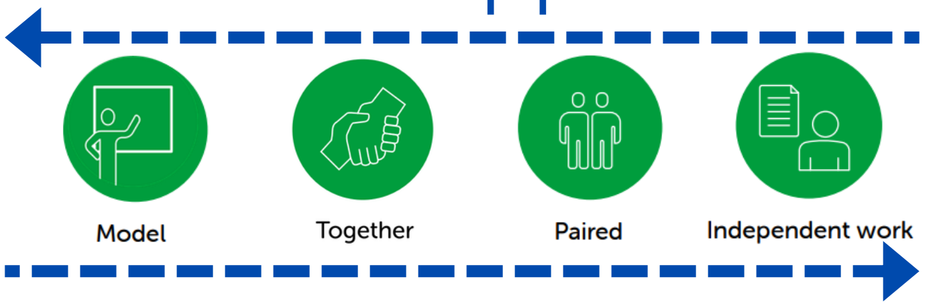


Concept Cartoons

Problem-Solving Metacognitive Process

	Plan	Monitor	Evaluate
Question	What is the key information? What am I trying to work	How am I going about it? Is the approach that I'm taking working?	What worked well when solving this problem? What didn't work well?
Clarify	Have I found all of the key information? Have I considered what is not there?	Do my answers at each stage look right? Do I know what I am going to do with them next?	Does my final answer look right? Does it make sense when I reread the question?
Visualise	What written methods can I see as valuable for this? Could I draw a model or picture to help?	Is what is in front of me helping answer the question? Have I used the method correctly?	Can I use an inverse method to check my answer?
Connect	What patterns, relationships or connections have I noticed?	Can I do anything with what I have found or noticed?	Does my answer fit with the connections? Have I missed anything to connect to?

Different Approaches



We provide pupils with the opportunity to approach problems in different ways:

1) slow release of responsibility - model, together, paired, you try can be used to enable children to see how a particular problem within a particular context can be solved. Staff should also consider modelling problems with similar structures yet different contexts to what they want pupils to attempt, thus allowing for metacognitive thinking and reasoning. An example of this could be modelling a solution to an area problem whilst offering pupils a volume problem with a similar structure. (EEF guidance recommendations 3 and 5 - KS2 Maths)

2) starting with a problem - independent, paired, model (if required) can be used to provide pupils with opportunities to experience 'desirable difficulty' and use their metacognitive skills ahead of having a chance to use their oracy skills to reason with a partner. (EEF guidance recommendation 5 - KS2 Maths)



True or False Reasoning Problems



Hedging

TRUE OR FALSE

These can be used to allow pupils the opportunity to consider statements, working, equations, conjectures etc. and allow them opportunity to explore key details before coming to a reasoned conclusion. (EEF guidance recommendation 3 - KS2 Maths)

HEDGING

These activities are designed to offer pupils greater opportunities for exploration of maths and verbal reasoning. Activities such as 'Which one doesn't belong?' and 'Could be, must be, can't be' are effective models of hedging. (EEF guidance recommendation 3 - KS2 Maths)

Both of the above should be used to develop pupil's reasoning skills and should be included with regularity - it is recommended they are used alongside our oracy talk tactics.

Goal-Free Problems

GOAL-FREE PROBLEMS

One of the findings from research about problem-solving is that, when presented with a problem, novices tend to use a strategy called "means-ends analysis" which involves thinking about the end-point (i.e. solution) and trying to work backwards to figure out where to start. This places a "load" on our thinking, because now we're having to process:

1. The end-point
2. The starting-point
3. How to get from the starting-point to the end
4. The information from the question (e.g. the diagram, values, text etc.)

This cognitive load is significant and often too much, resulting in students feeling overwhelmed and displaying opt-out behaviours like giving up and visible signs of frustration.

By removing the specific goal, we create a scenario where we can challenge pupils to find what they can thus eliminating the "means-end" issue. They now only have to process:

1.The starting point
2. The information from the question

This removes the requirement of a) having to figure out the end-point, b) having to find paths to the end-point, and c) having to check whether the working we've done has got us any closer to the end-point. In doing so, it lowers the cognitive load and means we can focus more on the concepts we do know and how we might use them, rather than the answer we don't know and how we might find it.

We want pupils to have the opportunity to "fill in the gaps" as opposed to being told to look into something specific.

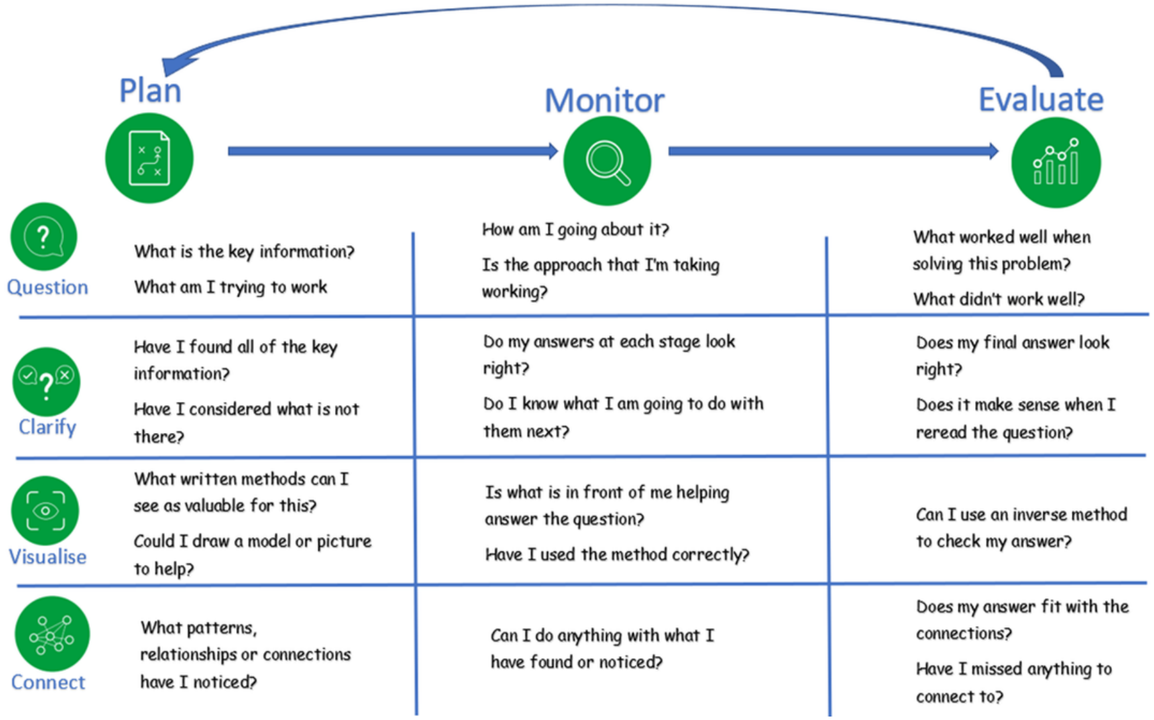
Worked examples: right and wrong

Using worked examples, or 'solved problems', present the problem and a correct solution together, they remove the need to carry out the procedures required to reach the solution and enable pupils to focus on the reasoning and strategies involved. Worked examples may be complete, incomplete, or incorrect, deliberately containing common errors and misconceptions for learners to uncover. Analysing and discussing worked examples helps students develop a deeper understanding of the logical processes used to solve problems. Use worked examples to enable pupils to analyse the use of different strategies. (EEF guidance recommendation 3 - KS2 Maths)

Concept Cartoons

Concept Cartoons can be used to provide pupil's opportunities to use and compare different approaches - there are often multiple ways to approach a problem. Much can be learned by examining different solutions to the same problem and looking for similarities in solution approaches to different problems. Pupils will need to distinguish between superficial similarity (for example, two problems both about carrots) and deeper similarities, relating to mathematical structure and approach. Teach pupils to interrogate and use their existing mathematical knowledge to ascertain which solutions are appropriate. Pupils should be encouraged to search their knowledge of similar problems they have encountered for strategies that were successful, and for facts and concepts that might be relevant. (EEF guidance recommendation 3 - KS2 Maths)

Problem-Solving Metacognitive Process



INVESTIGATION METACOGNITIVE PROCESS

This model is based on key findings from several EEF guidance reports, our own research, collaboration and pupil voice.

A problem-solving strategy is a general approach to solving a problem. The same general strategy can be applied to solving a variety of different problems. For example, a useful problem-solving strategy is to identify a simpler but related problem. Discussing the solution to the simpler problem can provide insight into how the original, harder problem may be tackled and the underlying mathematical structure - can a child see the problem as if it were an arithmetic question?
This 'toolkit' is designed to allow pupils to do just that.

Clarify, Question, Visualise, Connect are elements we have taken from the EEF guidance report for improving literacy at KS2 - recommendation 3. It clearly establishes that these strategies are essential when teaching and modelling comprehension. If we see these as key thinking skills, we can apply them to problem solving and reasoning. This idea is further supported in the EEF guidance report for KS2 maths - recommendations 3, 4 and 5, where pupils should be encouraged to use ***visual*** *representations; use* ***questions*** *to support their thinking and* ***clarify*** *their approach, whilst making appropriate* ***connections*** *within the maths and problem.*
Acting as the core frame to this 'toolkit' is metacognition and the three steps (plan, monitor, evaluate) involved within this process. Specifically, recommendations 2, 3 and 5 were used as the key basis.
Recommendation 2: Explicitly teach pupils metacognitive strategies, including how to plan, monitor, and evaluate their learning and this is directly linked to recommendation 5 in the KS2 maths guidance too.
Recommendation 3: Model your own thinking to help pupils develop their metacognitive and cognitive skills - this toolkit should be used by teachers to do exactly this and should be referred to when thinking aloud during the modelling process.
Recommendation 5: Promote and develop metacognitive talk in the classroom - this toolkit should be used with our oracy talk tactics to allow children to utilise and verbalise the metacognitive process.

Small Steps for Metacognitive Problem Solving

HOW DO WE DO IT?



Step 1: You are the expert and begin by modelling the metacognitive approach through modelled 'think alouds', utilising the problem solving toolkit as a clear and organised approach. The teacher should read the prompts aloud and then actively answer them out loud whilst modelling the process on the board. Pupils should have a copy of the toolkit and they should follow your steps as you use it in your 'think aloud'. This part of the process is key to show that we need to consider the problem in steps and as a whole as well as clarifying that we do not need to be preoccupied with the 'answer'. We should be doing this part of the process regularly and for a longer period of time before moving onto step 2.



Step 2: This builds on step 1 carefully. With this part of the process, pupils are actively involved by using the toolkit to question the teacher, using the toolkit prompts to enable them to work through the metacognitive steps, without worrying about actually ***'doing the maths'*** *themselves. The teacher is to respond to the pupils questions as if they were their own thoughts. They should answer them aloud, model what they would do on the board and respond with positive feedback to the pupils for asking appropriate questions. If pupils move to the monitor phase too early or ask questions that are not pertinent, the teacher should guide them appropriately.*



Step 3: Using their oracy framework talk tactics, coupled with the metacognitive process mapped out in the toolkit, the children should ask the questions to encourage their partner to respond in the context of the given problem. It is the teacher's responsibility to decide if one child takes the role of the 'questioner' and the other the 'responder or solver' or whether they can rotate roles to solve it together. Either way, the children must be given opportunities to take on both roles. Again, this step must be done over a period of time before moving onto Step 4, where independent practice begins.



Step 4: Using their metacognitive steps, the children are now ready to move independent practice. This should essentially involve them mimicking the teacher's 'think aloud' process from Step 1. They are in control of their own investigation here and need to use metacognition and critical thinking.

Variation remains key and the teacher needs to choose routine and non-routine investigations for the children to approach. We must ensure children do not become complacement through only seeing routie problems that directly fit with lesson content.

The Mathematical Thinker

A thinking, talking, doing and reflecting cycle for problem solving in maths.

THINK → TALK → DO
CHECK → REFLECT

1 THINK
Understand & Plan (Polya)

What is the problem asking?
What do I already know?
What could help?
Have I seen something like this before?

2 TALK
Explain & Reason (Duck Method + Oracy)

Explain it to the duck.
Say your thinking aloud.
Why does that work?
Convince someone.

3 DO
Try & Represent (Polya)

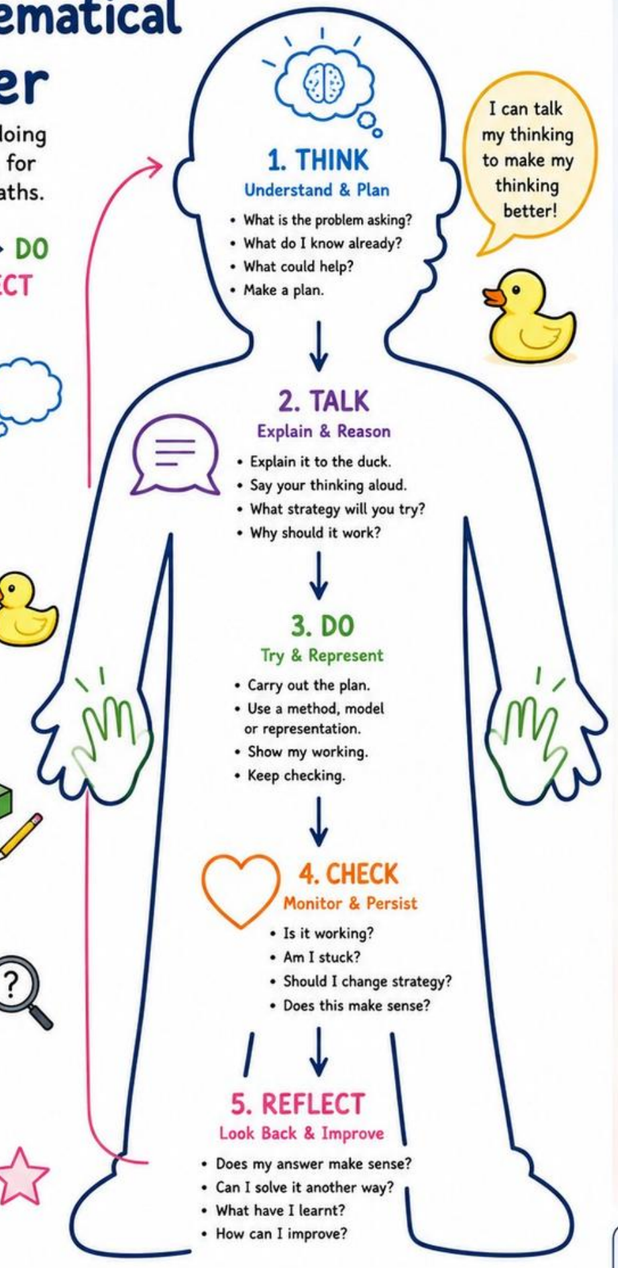
Carry out the plan.
Use a strategy or a model.
Show your thinking.
Keep checking as you go.

4 CHECK
Monitor & Persist (EEF Metacognition)

Is my strategy working?
Am I stuck?
Should I try something different?
Does this make sense?

5 REFLECT
Look Back & Improve (Polya + EEF Evaluation)

Does my answer make sense?
Can I prove it another way?
What have I learnt?
What will I remember next time?



At the heart of it all:
Curiosity • Perseverance • Resilience • Talk • Reasoning • Making Sense

EEF: 7 STEPS TO RELEASE OF RESPONSIBILITY

How support is gradually removed as learners take more ownership.

1. **I DO** — Teacher models and thinks aloud.
2. **WE DO** — Teacher and pupils work together.
3. **YOU DO TOGETHER** — Pupils work in pairs or small groups.
4. **YOU DO WITH HELP** — Pupils work independently with scaffolds.
5. **YOU DO ALONE** — Pupils work independently without support.
6. **YOU DO & TEACH** — Pupils explain and support others.
7. **YOU DO & REFLECT** — Pupils evaluate, reflect and take ownership of their learning.

★ The goal: Independent, confident mathematical thinkers who can explain, solve, check and improve.